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Influence of Mach Number on Coherent Structure Relevant to Jet Noise

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The cross-spectral densities of turbulent pressure and its azimuthal constituents have been obtained experimentally using condenser microphones in a round jet for the entire region between $1 \leq x/D \leq 12$ and $0 \leq r/D \leq 2$, and for range of Mach numbers $0.30 \leq M_0 \leq 0.70$. The coherent structures which were thus determined appear not to be strongly influenced by Mach number. A comparison of our measured phase velocities, as calculated from the axial distributions of phase of cross-spectral density, with those of other authors suggests a difference in eddy-wave behavior between acoustically-forced and unforced jets.

Nomenclature

a_0	= speed of sound
c_{ph}	= phase velocity in axial direction
D	= jet-nozzle diameter
f	= frequency
M_0	= U_0/a_0, Mach number
m	= number of azimuthal Fourier mode
$\tilde{p}^2_\omega$	= power-spectral density of turbulent pressure
r	= radial distance from jet axis
S_ω	= absolute value of W_{p1p2}
$S_{\omega,m}$	= absolute value of $\tilde{W}_{12,m}$
St	= $f \cdot D/U_0$, Strouhal number
U_0	= jet-exit velocity
W_{p1p2}	= cross-spectral density of turbulent pressure
$\tilde{W}_{12,m}$	= azimuthal constituents of W_{p1p2}
x	= axial coordinate along jet axis
ρ_0	= mean fluid density at center of jet
ϕ	= azimuthal angle
ψ_ω	= phase angle of W_{p1p2}
$\psi_{\omega,m}$	= phase angle of $\tilde{W}_{12,m}$
ω	= angular frequency

Introduction

SINCE 1963, when Mollø-Christensen[23] observed large-scale coherent structures in jets, these structures have been the subject of intense investigation by several authors. Mollø-Christensen's findings were based on his measurements with microphones near a jet, just outside the jet flowfield. Lau et al.[17] using specially adapted microphone probes in the potential core of a subsonic jet, were able to further verify the existence of a relatively periodic and quasideterministic flow structure inherent in that which formerly was assumed to be completely stochastic turbulence. Fuchs[12] showed, additionally, that this structure is particularly strong for certain frequencies of narrow-band filtered-pressure signals. These findings have led to some doubt about the eddy model of turbulence, which is most widely used in theories of jet noise.

Parallel to the development of the knowledge of turbulence structure in jets, Michalke[21,22] modified the Lighthill[19] theory of aerodynamic noise generation such that the source term is a function of the pressure fluctuations in the flowfield and their cross-spectral densities. These can be measured with a reasonable degree of accuracy and limited experimental effort in contrast to the Reynolds stresses, which are the basis of the source term in Lighthill's analysis. Furthermore, Michalke takes account of the axial symmetry of a circular jet in that he describes the cross-spectral density of the noise sources in cylindrical coordinates. The turbulent pressures are resolved into a Fourier series of azimuthal constituents in the circumferential angle of the jet and it is found that the lower order azimuthal constituents can be efficient in generating noise. Michalke and Fuchs[22] then presented experimental data which show that more than 85% of the turbulent energy can be contained in the first four azimuthal constituents. This is in rather strong contrast to the small-scale eddies on one hand and the annular-vortex models of jet turbulence according to Hardin[13] and Laufer et al.[18] on the other. The latter assume the turbulent energy to be concentrated in the large torroidal vortices, which correspond to just the axisymmetric-turbulent mode.

In conclusion, it can be said that the turbulence model of azimuthal-frequency components is most compatible with the geometry and turbulence structure of a round jet as revealed by the pressure-correlation techniques. Also the completeness of other models can be questioned somewhat, since there is not yet any method using these models with which jet noise can be calculated in a strictly quantitative manner.

It was the purpose of the following study to obtain the experimental data necessary for Michalke's jet-noise model and, in particular, to determine the influence of the Mach number M_0 on the coherent structures in the range of $M_0 = 0.30$ to 0.70. It was assumed that the variation of Reynolds number by changing the jet velocity is of minor importance for the structure of turbulence.

Preliminary tests were required to verify the reliability of microphone probes for measuring turbulent pressure fluctuations in the mixing region of a jet at high-subsonic Mach numbers. The preliminary work is described by Armstrong.[2] The theoretical fundamentals have already been adequately outlined by Michalke and Fuchs[22] and by Armstrong et al.[3]

Coherent Structure in a Circular Jet

The measurements were made in an air jet emitted by a circular nozzle of diameter $D = 5$ cm, which was supplied continuously by a radial compressor. Data presented here was obtained with standard Bruel & Kjaer 0.3175 cm condenser microphones fitted with nose cones. Pressure signals from these microphones were analyzed completely on a Hewlett-Packard 2116C digital computer. For details of the test setup, instrumentation and data processing, the reader may refer to Armstrong[2] and Armstrong et al.[3]

The characteristic regions of a round jet are shown illustratively in Fig. 1. The so-called potential core of the jet extends to about $x/D = 5$, in which region the vorticity is zero and the mean velocity remains constant. Thereafter the mean axial velocity decreases along the jet axis with increasing x/D. The actual outer boundary is locally very irregular and varies randomly with time. The drawn jet border is a time average of

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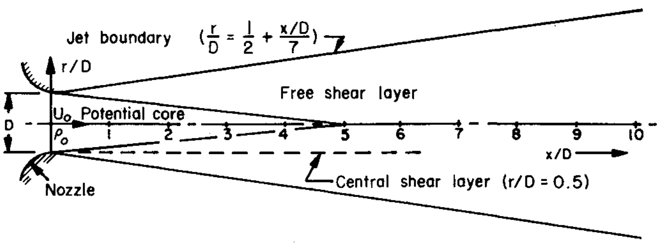

Fig. 1 Characteristic flow regions of a turbulent round jet.

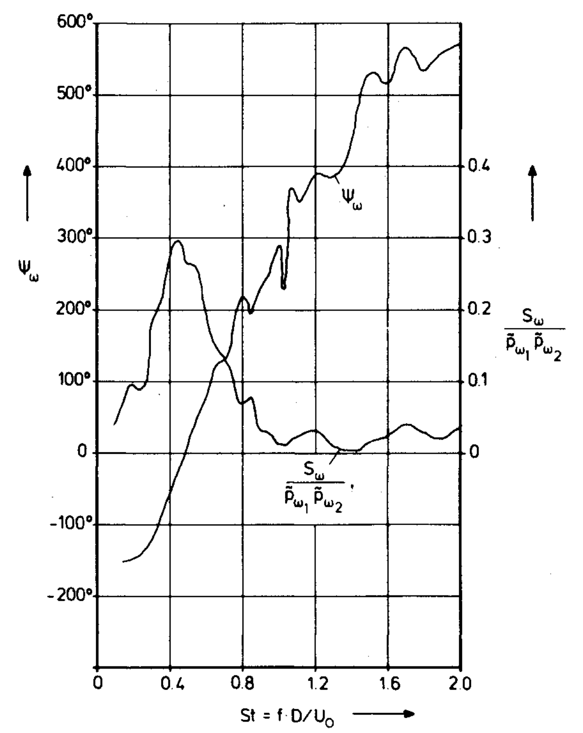

Fig. 2 Spectra of the absolute value and phase of CSD in a jet with $x_1/D=3$; $x_2/D=4.5$; $r_1/D=0.5$; $r_2/D=1.0$; $\Delta\phi=90$ deg; $D=5$ cm; $M_0=0.50$; $\Delta St=0.031$.

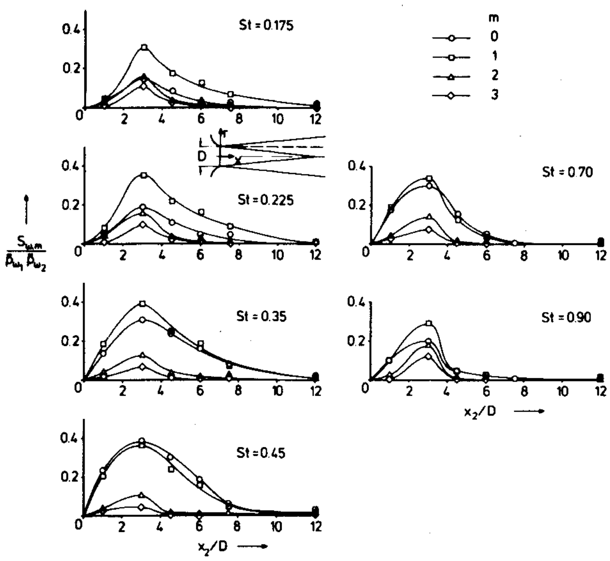

Fig. 3 Axial distribution of the absolute value of azimuthal components of CSD of pressure fluctuations at $r/D=0.5$; $M_0=0.50$; $x_1=3D$; $r_1=r_2=r$; $D=5$ cm; $\Delta St=0.031$.

the position of the actual edge of the jet and is in this case defined as the location where the intermittence factor $\gamma=0.5$. The time average of the border widens linearly with increasing x approximately according to the law $r/D=(1/2+1/7)x/D$ (cf. Komatsu[15]). Between edge and potential core of the jet, turbulence develops in the free shear layer. There, the axial velocity, which at the edge of the potential core is equal to the velocity on the jet axis, becomes smaller with increasing distance from the axis until zero velocity is reached at the edge of the jet.

According to more recent findings of Crow and Champagne,[9] Lau and Fisher,[16] and Brown and Roshko[5] it appears that vortices of the order of magnitude of the shear layer thickness are also generated by an instability in the turbulent free shear layer. They move downstream, pair off, and finally disintegrate. By this turbulent vortex generation a relatively coherent structure of the turbulent field arises in the range of the vortex frequencies. After about 25 nozzle diameters from the nozzle exit the jet becomes fully turbulent; the bell-shaped velocity profiles become similar at all cross-sections, and the intensity of the axial velocity fluctuations becomes practically independent of radius in the central region of the jet (cf. Corrsin[8]); after about 70 nozzle diameters from the nozzle exit the radial profiles of turbulence ($\tilde{u}/U_{max}$) also become similar (cf. Wygnanski and Fiedler[24]).

In the present test the Mach number was usually $M_0=0.50$, the highest (local) Mach number for which the microphone probe could be shown not to be considerably disturbing the jet pressure field. A typical result of a cross-spectral density (CSD) measurement is shown in Fig. 2, a spectral distribution of the normalized absolute value of CSD $S_\omega/\tilde{p}_{\omega 1}\tilde{p}_{\omega 2}$ and phase ψ_ω of the pressure fluctuations for the Strouhal number region from $St=0.10$ to 2.0. In this case the measuring probes were located at $x_1/D=3$, $x_2/D=4.5$, $r_1/D=0.50$, $r_2/D=1.0$ and the circumferential displacement angle was $\Delta\phi=90$ deg. The normalized absolute value has a maximum ($\simeq 0.3$) at $St=0.43$, which indicates that the pressure fluctuations at probe locations 1 and 2 are especially strongly coherent at $St\simeq 0.45$. The phase relationship between the pressure fluctuations is approximately a linear function of the Strouhal number. The CSD was determined with the second microphone at various radial positions. In this series of measurements the first probe was always positioned at $r_1/D=0.50$ in the middle of the shear layer, three nozzle diameters downstream of the nozzle where the pressure fluctuations at the Strouhal number $St=0.45$ are strongest. The frequency components of the CSD were further resolved into azimuthal constituents by means of circumferential correlations in a manner described by Michalke and Fuchs.[22]

A typical example of an axial distribution of the normalized azimuthal-frequency components of the CSD $S_{\omega,m}/\tilde{p}_{\omega 1}\tilde{p}_{\omega 2}$ (where m is the number of the azimuthal-Fourier mode) with the second probe also in the middle of the shear layer at $r_2/D=0.50$ is shown in Fig. 3 for the Strouhal number range from $St=0.10$ to 0.90. Only the $m=0$, 1, 2, and 3 azimuthal constituents are shown, because the turbulent energy of the azimuthal constituents declines very rapidly with increasing azimuthal number m, leaving as much as 95% of the energy in the first four constituents. All of the normalized azimuthal-frequency components $S_{\omega,m}$ have a maximum at $x_2/D=3$ as one would expect. The axisymmetric $m=0$ and the first azimuthal $m=1$ constituents dominate over the entire Strouhal number range studied, especially at $St=0.45$.

The radial distribution of the azimuthal-frequency components of the CSD with $x_1/D=x_2/D=3$, where the frequency component $St=0.45$ of the pressure intensities is greatest, is given in Fig. 4 for the Strouhal numbers $St=0.10$ to 0.90. In the entire Strouhal number range studied the axisymmetric constituent of the CSD is, due to symmetry, the only constituent which is nonzero at $r=0$, complying with $S_{\omega,m}\sim r_2^m$ (cf. Michalke[21]). The normalized axisymmetric constituent $S_{\omega,0}$ is greater at $r_2=0$ than at $r_2/D=0.5$ because $\tilde{p}_{\omega 2}$ has a minimum at $r_2=0$. In this respect normalizing the azimuthal-frequency components of the CSD with $\tilde{p}_{\omega 1}$ and $\tilde{p}_{\omega 2}$ proves not to be optimal. The $m=1$ component has the highest rate of increase with r_2 in the vicinity of the jet axis.

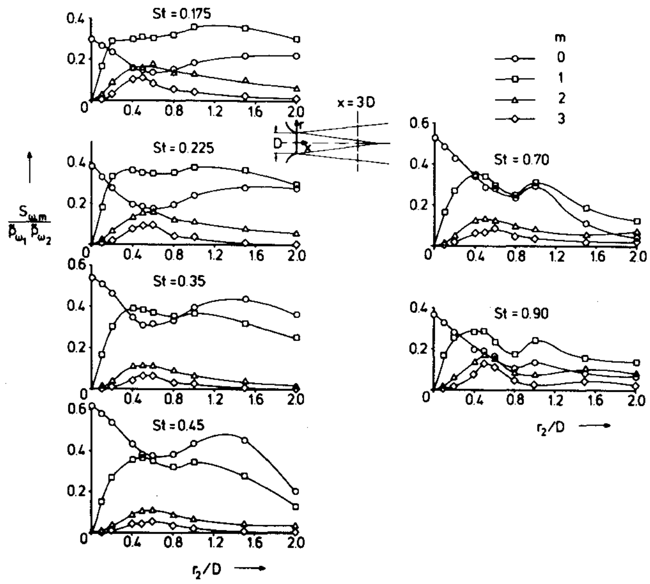

Fig. 4 Radial distribution of the absolute value of azimuthal components of CSD of pressure fluctuations normalized with local rms pressures at $x/D=3$; $D=5$ cm; $x_1=x_2=x$; $r_1/D=0.5$; $M_0=0.50$; $\Delta St=0.031$.

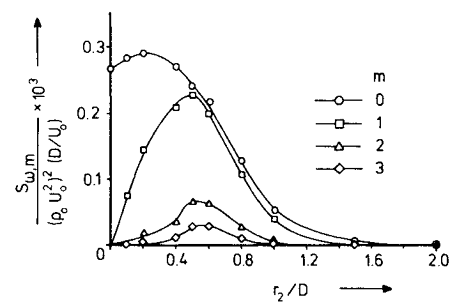

Fig. 5 Radial distribution of the absolute value of azimuthal components of CSD of pressure fluctuations normalized with nozzle exit plane parameters. $M_0=0.50$; $x_1/D=x_2/D=3$; $r_1/D=0.5$; $St=0.45$.

In the middle of the Strouhal number range, $St=0.35$ to 0.70, in which the pressure intensities are greatest, and for $r_2/D>0.30$, the first two constituents are of the same order of magnitude and much larger than the higher-order constituents. This supplements the findings in the distributions of the CSD with axially-displaced probes and shows that it is the $m=0$ and 1 constituents which are the most prominent ones. In the entire Strouhal number range $St=0.175$ to 0.90 the normalized axisymmetric constituents have minima in the jet-mixing region between $r_2/D=0.5$ and 0.8 which occur at larger radii for increasing Strouhal number. The normalized $m=1$ constituents of CSD also have minima at $r_2/D=0.8$ at the edge of the mixing zone. These minima, however, do not indicate that there are minima in the distribution of the CSD if they are not normalized with the pressure $\tilde{p}_{\omega 1}\tilde{p}_{\omega 2}$. An example of the radial distribution of the azimuthal-frequency components of CSD $S_{\omega,m}$, normalized with the jet nozzle discharge plane parameters $(\rho_0 U_0^2)^2(D/U_0)$ is shown in Fig. 5 for $St=0.45$. It can be seen that the CSD's normalized in this manner have a very regular distribution. All four components have maximum intensities between $r_2/D=0.2$ and 0.6, which is within the mixing region.

At most Strouhal numbers the constituents $m=0$ and 1 of $S_{\omega,m}/\tilde{p}_{\omega 1}\tilde{p}_{\omega 2}$ remain large far beyond $r_2/D=0.8$ (cf. radial distribution, Fig. 4), outside the jet where the mean-flow velocity is already zero. Further axial and radial distributions of the cross-spectral density of the pressure fluctuations in a jet are reported by Armstrong.[2]

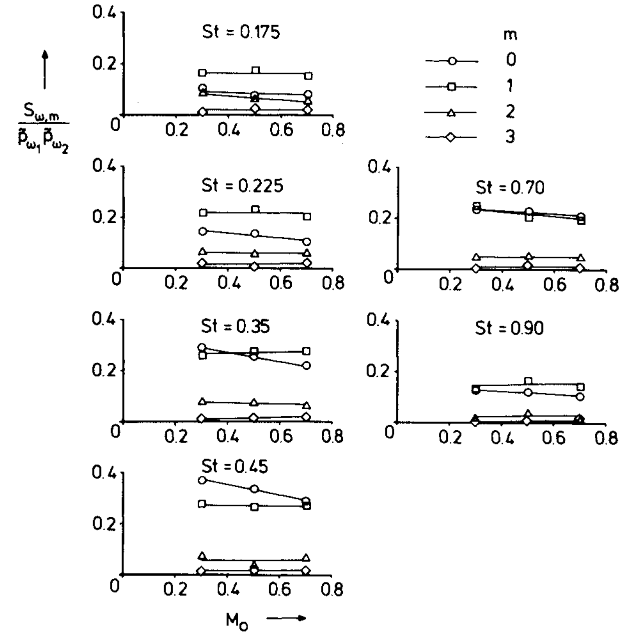

Fig. 6 Influence of Mach number on normalized-absolute value of azimuthal-frequency components of CSD. $x_1/D=3$; $r_1/D=0.5$; $x_2/D=1.0$; $r_2/D=1.0$; $D=5$ cm; $\Delta St=0.031$.

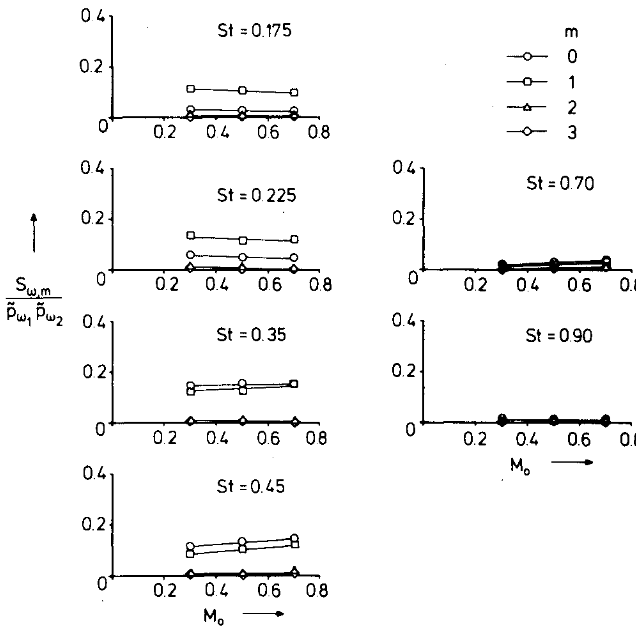

Fig. 7 Influence of Mach number on normalized absolute value of azimuthal-frequency components of CSD. $x_1/D=3$; $x_2/D=6$; $r_1/D=0.5$; $r_2/D=1.0$; $D=5$ cm; $\Delta St=0.031$.

Influence of Mach Number on Coherent Structure

The influence of Mach number on azimuthal-frequency components of coherence was studied with the second measuring probe at $x_2/D=1$, $r_2/D=1$. Hence, the probe was near the jet nozzle and outside of the jet; however, it was well within the pressure near field. The reference microphone was located at $x_1/D=3$, $r_1/D=0.5$. The coherence is plotted in Fig. 6 for the Mach numbers $M_0=0.3$, 0.5, and 0.7 and for the same Strouhal number range as before. With increasing Mach number the axisymmetric constituents become somewhat smaller at all Strouhal numbers, whereas the constituents $m=1$, 2, and 3 remain quite constant. It can be concluded, therefore, that the Mach number hardly has an influence on the degree of coherence or order in the jet turbulence. This tendency is not influenced by the fact that the

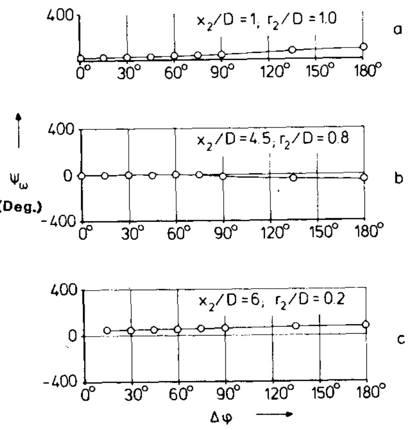

Fig. 8 Dependence of phase of CSD on circumferential-angle separation at $x_1/D=3$; $r_1/D=0.5$; $M_0=0.50$; $St=0.45$; $\Delta St=0.031$; $D=5$ cm.

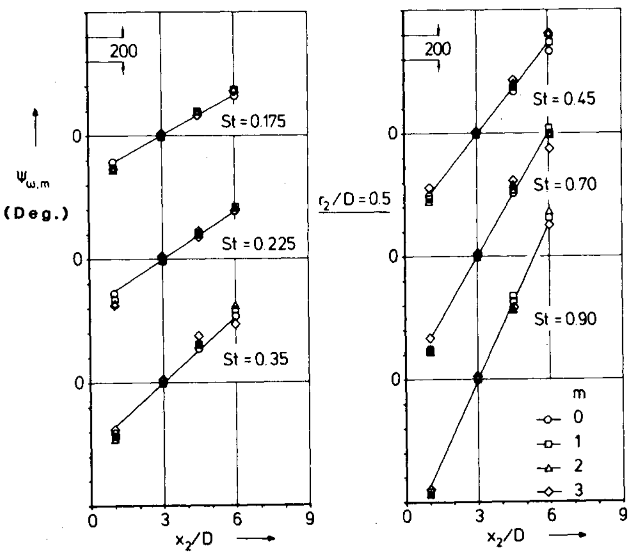

Fig. 9 Axial distribution of phase of azimuthal-frequency components of CSD at $x_1/D=3$; $r_1/D=r_2/D=0.5$; $M_0=0.50$; $\Delta St=0.031$; $D=5$ cm.

filtered pressure intensities ($\bar{p}_\omega$) increase with increasing Mach number (cf. Armstrong, Michalke, and Fuchs[3]), for the cross-spectral densities are normalized with the pressure fluctuations $\bar{p}_{\omega 1}$ and $\bar{p}_{\omega 2}$. More specifically, these results confirm the persistence of coherent structures at high subsonic Mach numbers.

The influence of the Mach number on coherence was also studied with the second probe at another position. Results with the second probe at $x_2/D=6$, $r_2/D=1$, downstream of the reference probe in the outer part of the mixing zone, are shown in Fig. 7. The measurements were made at the Mach numbers $M_0=0.3$, 0.5, and 0.7 at the same Strouhal numbers as before. The constituents of cross-spectral density are smaller than in the previous study (Fig. 6) at all Strouhal numbers. It can be seen that for increasing Mach number at the smaller Strouhal numbers, $St=0.175$ to 0.225, the constituents $m=0$ and 1 become somewhat smaller, and at the higher Strouhal numbers, $St=0.35$ to 0.90, they become somewhat larger or remain constant. The constituents $m=2$ and 3 were zero in this case. This study verified the results of the first one in that it confirms that the existence of large-scale coherent structures is by no means restricted to any particular fluid dynamic range, such as low Mach or Reynolds numbers, for example.

Phase of Cross-Spectral Density of Pressure Fluctuations

The circumferential distribution of the phase ψ_ω of CSD was studied in detail. Several examples of the distributions of ψ_ω as a function of $\Delta\phi$ are shown in Figs. 8a-c for three

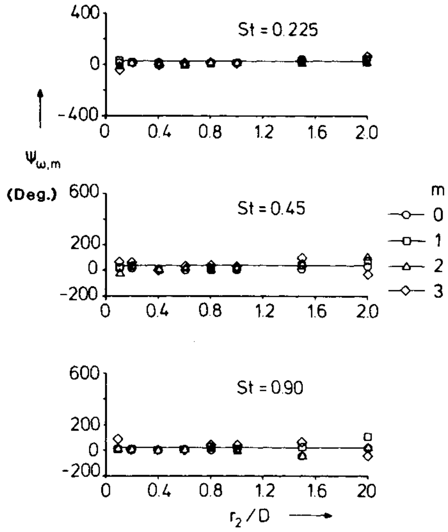

Fig. 10 Radial distribution of phase of azimuthal-frequency components of CSD at $x_1/D=x_2/D=3$; $r_1/D=0.5$;; $M_0=0.50$; $\Delta St=0.031$; $D=5$ cm.

different locations in the jet at a representative Strouhal number $St=0.45$. It can be seen that, relative to the axial dependence (Fig. 9), the phase is only slightly dependent on $\Delta\phi$; however, the existing circumferential variations of ψ_ω might indicate that there is a preferred helical propagation of pressure disturbances.

The axial distribution of the phase of azimuthal-frequency components $\psi_{\omega,m}$ of CSD is shown for the azimuthal constituents $m=0$, 1, 2, and 3 in Fig. 9. Both microphone probes were in the middle of the shear layer at $r/D=0.5$. The distribution of phase in the Strouhal number range $St=0.175$-0.90 is approximately the same for all azimuthal constituents within the measuring accuracy, and thus practically independent of m. The phase $\psi_{\omega,m}$ is linear in x. Therefore, for a given Strouhal number, the phase velocities for all azimuthal constituents of the pressure fluctuations $c_{\rm ph}$, which is defined here as

$$c_{\rm ph}=\frac{\omega}{\partial\psi_{\omega,m}/\partial x_2} \tag{1}$$

are approximately equal and independent of x. Without showing supporting data it is remarked here that the axial distributions of $\psi_{\omega,m}$ are identical with those of ψ_ω, thus phase velocities of azimuthal constituents are equal to phase velocities of frequency components which are not resolved into azimuthal constituents. This corresponds to the result that ψ_ω is nearly independent of $\Delta\phi$, for using our definition of CSD,

$$W_{p1p2}=(1/2)S_\omega\exp(i\psi_\omega)=1/2\sum_{m=0}^{\infty}S_{\omega,m}\exp(i\psi_{\omega,m})\cos m\Delta\phi \tag{2}$$

it can be shown that

$$\psi_\omega=\arctan\frac{\sum_m S_{\omega,m}\sin\psi_{\omega,m}\cos m\Delta\phi}{\sum_m S_{\omega,m}\cos\psi_{\omega,m}\cos m\Delta\phi} \tag{3}$$

which indicates that ψ_ω is independent of $\Delta\phi$ when $\psi_{\omega,m}$ is independent of m.

The radial distribution of the phase of the azimuthal-frequency components are shown in Fig. 10 for the case when both probes are at $x/D=3$. The phase distributions are

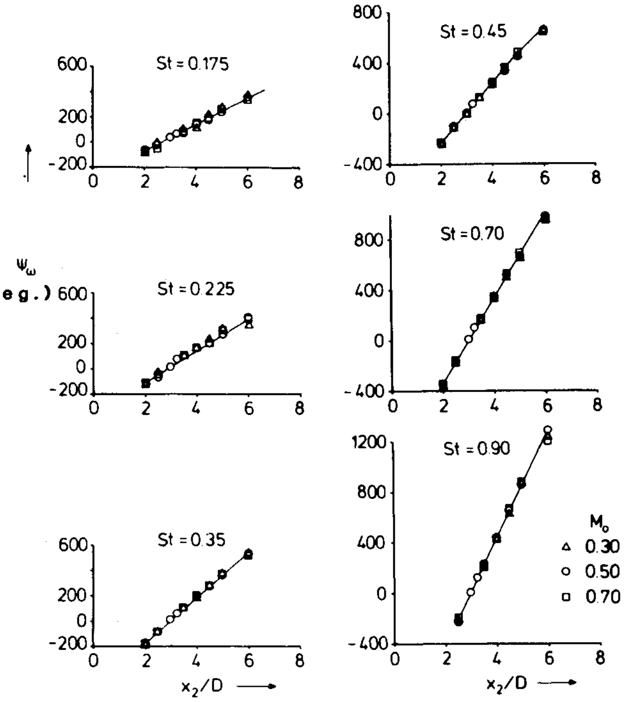

Fig. 11 Axial distribution of phase of CSD at several Mach numbers. $x_1/D=3$; $r_1/D=r_2/D=0.5$; $\Delta\phi=90$ **deg;** $\Delta St=0.031$; $D=5$ **cm.**

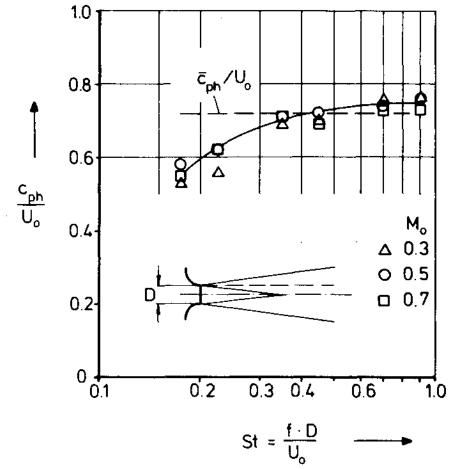

Fig. 12 Spectral distribution of phase velocities in the central shear layer of a jet at several Mach numbers. $x_1/D=3$; $r_1/D=r_2/D=0.5$; $2\leq x_2/D\leq 6$; $\Delta St=0.031$; $D=5$ **cm.**

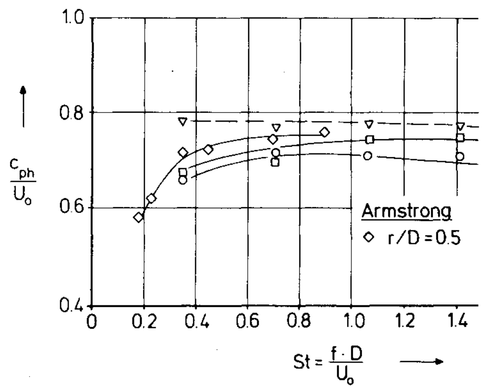

Fig. 13 Spectra of phase velocities compared to those of Ko and Davies.[14]

practically independent of m and constant with r. The scatter in the data seems to be a question of the measuring accuracy.

The same tendencies hold true for the phase of the frequency components ψ_ω, not resolved into azimuthal constituents, and for $\psi_{\omega,m}(r)$ at other axial locations in the jet according to further tests. Therefore, it can be concluded that the phase is practically independent of r. On the other hand, we recall that the amplitude of these structures varies very strongly in the radial direction (cf. Fig. 5). Thus the average spacetime structure of turbulence in the jet near field is dominated by a downstream propagation of relatively simple, large-scale structures.

Influence of Mach Number on Phase and Phase Velocity

Since the phase ψ_ω and $\psi_{\omega,m}$ of the frequency components of pressure are approximately independent of circumferential angle $\Delta\phi$, and from azimuthal number m, respectively, and from radius r, the influence of Mach number on phase was studied only for its axial distribution. The phase ψ_ω is plotted over x_2/D for the Mach numbers $M_0=0.30$, 0.50, and 0.70 in the Strouhal number range $St=0.175$-0.90 in Fig. 11. Both probes were in the central mixing region at $r/D=0.5$ and $\Delta\phi=90$ deg. It can be seen that at each Strouhal number the phase ψ_ω is completely independent of Mach number. Therefore, the gradient of the phase over x_2/D and, thus, also the phase velocity is independent of Mach number in the range $0.30\leq M_0\leq 0.70$.

The axial-phase velocity of the pressure field can, as has already been shown, be calculated from the gradient $\partial\psi_\omega/\partial x_2$ of the axial distribution. c_{ph} is usually normalized with the jet exit velocity U_0. Thus, using Eq. (1) we obtain

$$\frac{c_{ph}}{U_0}=\frac{\omega}{U_0(\partial\psi_\omega/\partial x_2)}=\frac{\omega D}{U_0[\partial\psi_\omega/\partial(x_2/D)]}=\frac{2\pi St}{\partial\psi_\omega/\partial(x_2/D)} \tag{4}$$

The phase velocity in the mixing zone of a jet is plotted over Strouhal number in Fig. 12 for the Mach numbers $M_0=0.30$, 0.50, and 0.70. The values of c_{ph} are the results of phase measurements at a constant circumferential angle $\Delta\phi=90$ deg. As already implied in the above paragraph, the normalized-phase velocity is relatively independent of Mach number. The phase velocity increases with increasing Strouhal number, especially at lower Strouhal numbers. These values of c_{ph}/U_0 compare quite well with data published by some other authors; e.g. Fisher and Davies,[10] Ko and Davies,[14] and Ackermann.[1] However, the phase velocities do not agree well with those measured in forced jets by researchers such as Chan[6] and Crow and Champagne,[9] both of whom showed phase velocity decreasing with increasing Strouhal number.

Comparison of Results with Those of Other Authors

Although other authors who have done similar work have sometimes had different goals and/or used different measurement techniques, some of their results can be compared with ours.

Ko and Davies,[14] for example, calculated the local phase velocities of velocity fluctuations in a jet from cross-spectral densities of signals measured with two parallel hot-wires spaced 1.55 mm apart. If we assume that the phase velocities of the pressure fluctuations and those of the velocity fluctuations induced by the same vortical motion are equal, which is common practice, (cf. Fuchs[11]), then we can compare our test results with those of Ko and Davies. Our phase velocities at $r_2/D=0.5$ are plotted with those of Ko and Davies as a function of Strouhal number in Fig. 13. Our results are valid for the range $2<x/D<6$ where c_{ph}/U_0 was constant, whereas Ko and Davies measured different phase velocities at $x_2/D=2.5$, 3, and 3.5. Our results lie approximately in the middle of those phase velocities determined by Ko and Davies.

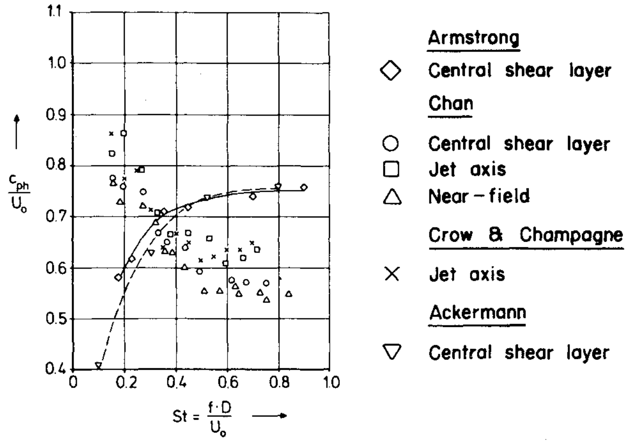

Fig. 14 Spectral distribution of phase velocity in a jet compared to those of other authors.

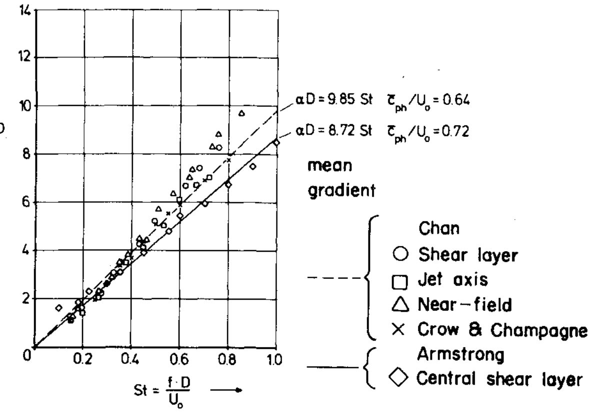

Fig. 15 Spectral distribution of wave number in a jet compared to those of other authors.

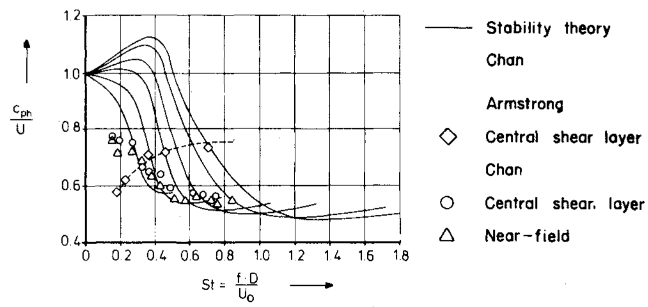

Fig. 16 Phase velocity, calculated from stability theory, compared with experimental data.

Ackermann[1] obtained phase velocities as we did from axial distributions of the cross-spectral density of pressure fluctuations measured with microphones in a jet. He used a nozzle with diameter $D = 7.5$ cm and a 0.635 cm B&K microphone; the jet velocity was about $U_0 = 60$ m/s. Chan[6] measured turbulent pressure fluctuations in the mixing region and near field of a jet excited with a loudspeaker. He also used a 0.635 cm condenser microphone with nose cone. He determined phase velocities from oscillographs of instantaneous spatial distributions of pressure signals at two positions on the jet axis. His nozzle diameter was $D = 5.715$ cm and jet velocity $U_0 = 67$ m/s.

Crow and Champagne[9] measured phase velocity of axial-velocity fluctuations on the axis of a jet excited with a loudspeaker. They used the hot wire technique in connection with spatial oscillograms as Chan did. Their nozzle diameter was $D = 2.54$ cm, and U_0 was about 60 m/s.

The spectral distributions of our normalized phase velocity c_{ph}/U_0 in the middle of the jet-shear layer, which are typical of our experimental results, are plotted with those of Ackermann, of Chan, and of Crow and Champagne over the Strouhal number for $St = 0.175$ to 0.90 in Fig. 14. Our phase velocities increase with increasing Strouhal number. Ackermann's measurements agree very well with ours. The experimental results of the other authors generally show the opposite tendency; namely, that phase velocity decreases with increasing Strouhal number. This radical difference in measured results appears to point to a difference in turbulence structure between an acoustically-forced and a nonacoustically-forced jet.

Furthermore, it was later implied by Chan[7] that a jet which is internally excited upstream of the nozzle will exhibit a primarily axisymmetric ($m = 0$) structure in which phase velocity decreases with increasing Strouhal number. His measurements at the later date in an externally excited jet with variable periodic forcing, which are not shown here, showed that for higher order modes ($m = 1,2$) phase velocity increases with increasing Strouhal number; therefore, the forcing method influences the structure that will predominate. Also, we know that, in an unforced jet like ours (cf. Armstrong et al.[3]), the energy of the sum of the $m = 1$, 2, and 3 modes must dominate over that of the $m = 0$ mode; thus, the phase velocity of the unforced jet should and does behave like that in Chan's externally forced jet.

Converse to Chan's findings of different phase velocity for different turbulent structures, Fig. 9, which has already been described, indicates that the $m = 0$, 1, 2, and 3 azimuthal components of pressure in an unforced jet all have approximately the same phase velocity. Thus, the present comparison of data seems to indicate that there is a basic difference between the behavior of pressure waves in an unforced jet and in an acoustically-forced jet.

The spectral distribution of dimensionless wave number αD calculated from phase velocity is not so sensitive to Strouhal number as phase velocity. The wave numbers corresponding to phase velocities in the middle of the jet-shear layer are therefore plotted with wave numbers measured by Chan[6] and by Crow and Champagne[9] in Fig. 15. In the middle Strouhal number region for $St = 0.2$ to 0.5 we see relatively good agreement between our results and those of other authors. At lower and higher Strouhal numbers deviations appear. The average phase velocity calculated from the published results of Chan and of Crow and Champagne (cf. dashed straight line) of $\bar{c}_{ph}/U_0 = 0.64$ is 11% smaller than our average phase velocity of $\bar{c}_{ph}/U_0 = 0.72$.

Chan[6] modified Michalke's stability theory for free shear layers such that the increasing thickness of the shear layer was taken into consideration. He subsequently calculated the spectral distribution of phase velocity for various axial distances from the nozzle exit plane. The spectra for $x/D = 0.5$ to 2.0 are plotted in Fig. 16. The envelope of these spectra represent the spectrum for continuously varying x/D. Our results of phase velocity in the central mixing region between $x/D = 2$ and 6 and the results of Chan[6] are included in the figure for comparison with the theory. Our measured spectral distribution of phase velocity runs diagonally across the theoretical curves and indicates a tendency opposite to that of the theory. On the contrary, the data of Chan agrees well with the envelope of the theoretical curves. This indicates that the results of linear stability theory are perhaps better applicable to the turbulence structure of an (internally) acoustically-forced jet than to that of an unforced jet.

Conclusions

The distribution of the fluctuation energy in the azimuthal constituents of turbulent pressure is practically independent of the Mach number throughout the jet near field. A dependence of phase on cross-spectral density or phase velocity on Mach number also could not be found. If it is assumed that an influence of Mach number on the physical spreading of a jet in the transverse direction is related to a change in large-scale turbulence structure, then our above results agree with those of Bradshaw.[4] He showed, namely, that between the Mach numbers $M_0 = 0.4$ and 0.7 there is practically no change in the shear layer width. Only the rms values of the pressures are slightly dependent on Mach number.

The test results present a very simple model which could, as a first approximation, describe the turbulence structure in a jet mixing layer. The pressure fluctuation field between $x = D$ and $8D$ fluctuates very coherently with frequencies around $St = 0.45$ (mainly $St = 0.175$ to 0.90) and consists for the most part only of axisymmetric and the first azimuthal constituents. The pressure fluctuations propagate downstream with a constant-phase velocity, which is approximately 70% of the jet exit velocity. Phase velocity is practically independent of the Strouhal number and is of the same magnitude for all azimuthal constituents. A Mach number dependence of the turbulence structure is, for a first order approximation, negligible in the limited range studied.

A comparison of our measured-phase velocities with those of other authors suggests a basic difference between the turbulence structure of acoustically-forced and that of unforced jets. Furthermore, the results of linear stability theory appear to be better applicable to pressure waves in (internally) forced jets than to those in unforced jets.

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